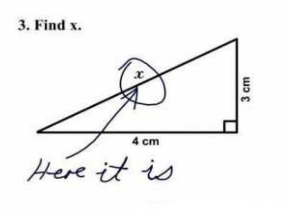
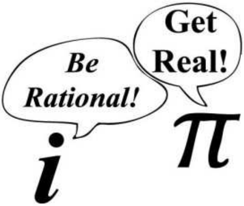
JU Mješovita elektrotehnička i
drvoprerađivačka srednja škola
Bihać

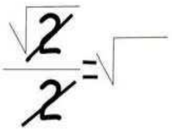
Matematika

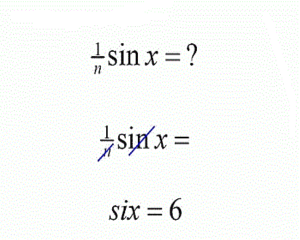
[interna skripta formula]

www.etsbi.edu.ba









'Imaj hrabrosti da se služiš sopstvenim razumom.'
Immanuel Kant [1724-1804]

autor - Mirzet Brkić
recenzent - Armin Jusić, prof.
ETSBI/p4v 2012.g.

Kompleksan broj:

$$i^2 = -1, z = a + bi, \bar{z} = a - bi$$

$$|z| = \sqrt{a^2 + b^2}; a, b \in \boldsymbol{R}$$

$$z = r(\cos\varphi + i\sin\varphi)$$

$$z_1 \cdot z_2 = r_1 r_2[\cos(\varphi_1 + \varphi_2) + i\sin(\varphi_1 + \varphi_2)]$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2}[\cos(\varphi_1 - \varphi_2) + i\sin(\varphi_1 - \varphi_2)]$$

$$z^n = r^n(\cos n\varphi + i\sin n\varphi),$$

$$\sqrt[n]{z} = \sqrt[n]{r}\left[\cos\left(\frac{\varphi + 2k\pi}{n}\right) + i\sin\left(\frac{\varphi + 2k\pi}{n}\right)\right]$$

$$k = 0, 1, \ldots, n-1$$

****

$$a^m \cdot a^n = a^{m+n} \qquad a^m : a^n = a^{m-n}; (a \neq 0)$$

$$a^{-m} = \frac{1}{a^m}; (a \neq 0) \qquad \sqrt[m]{a^n} = a^{\frac{n}{m}}$$

$$(a \pm b)^2 = a^2 \pm 2ab + b^2$$

$$(a \pm b)^3 = a^3 \pm 3a^2b + 3ab^2 \pm b^3)$$

$$a^2 - b^2 = (a - b)(a + b)$$

$$a^3 \pm b^3 = (a \pm b)(a^2 \mp ab + b^2)$$

$$(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{k} a^{n-k} b^k + \ldots +$$

$$+ \binom{n}{n-1} a b^{n-1} + b^n$$

Kvadratna jednačina:

$$ax^2 + bx + c = 0;\ a \neq 0$$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Vièteove formule:

$$x_1 + x_2 = -\frac{b}{a} \qquad x_1 \cdot x_2 = \frac{c}{a}$$

Tjeme parabole:

$$T\left(-\frac{b}{2a}, \frac{4ac - b^2}{4a}\right)$$

****

$$b^x = a \Leftrightarrow x = \log_b a$$

$$\log_b b^x = x = b^{\log_b x}$$

$$\log_b(xy) = \log_b x + \log_b y$$

$$\log_b \frac{x}{y} = \log_b x - \log_b y \qquad \log_b x^y = y \log_b x$$

$$\log_a x = \frac{\log_b x}{\log_b a}$$

Površina trougla:

$$P = \frac{a \cdot h_a}{2} \qquad P = \frac{ab \sin\gamma}{2} \qquad P = \frac{abc}{4R} \qquad P = rs$$

$$P = \sqrt{s \cdot (s-a) \cdot (s-b) \cdot (s-c)}$$

$$s = \frac{a+b+c}{2}$$

Jednakostranični trougao:

$$P = \frac{a^2\sqrt{3}}{4} \qquad h = \frac{a\sqrt{3}}{2} \qquad R = \frac{2}{3}h \qquad r = \frac{1}{3}h$$

Površina paralelograma:

$$P = a \cdot h_a$$

Površina trapeza:

$$P = \frac{a+c}{2} \cdot h$$

Površina i obim kruga:

$$P = r^2\pi \qquad O = 2r\pi$$

Površina kružnog isječka:

$$P = \frac{r^2\pi\alpha}{360}$$

Dužina kružnog luka:

$$l = \frac{r\pi\alpha}{180}$$

****

B =površina baze, M =površina omotača

H =dužina visine, r =poluprečnik

Zapremina prizme i valjka:

$$V = B \cdot H$$

Površina prizme i valjka:

$$P = 2B + M$$

Zapremina piramide i kupe:

$$V = \frac{1}{3} B \cdot H$$

Površina piramide:

$$P = B + M$$

Površina kupe:

$$P = M + B = r\pi(r + s)$$

Zapremina i površina kugle:

$$V = \frac{4r^3\pi}{3} \qquad P = 4r^2\pi$$

U pravouglom trouglu:

$$\text{sinus ugla} = \frac{\text{suprotna kateta}}{\text{hipotenuza}}$$

$$\text{kosinus ugla} = \frac{\text{nalegla kateta}}{\text{hipotenuza}}$$

$$\text{tangens ugla} = \frac{\text{suprotna kateta}}{\text{nalegla kateta}}$$

****

Sinusna teorema:

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$

Kosinusna teorema:

$$c^2 = a^2 + b^2 - 2ab\cos\gamma$$

****

$$\sin^2 x + \cos^2 x = 1 \qquad \operatorname{tg} x = \frac{\sin x}{\cos x}$$

$$\sin 2x = 2\sin x\cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$\sin(x \pm y) = \sin x \cos y \pm \sin y \cos x$$

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

$$\mathrm{tg}(x \pm y) = \frac{\mathrm{tg}\, x \pm \mathrm{tg}\, y}{1 \mp \mathrm{tg}\, x \cdot \mathrm{tg}\, y}$$

$$\sin x + \sin y = 2 \sin \frac{x + y}{2} \cos \frac{x - y}{2}$$

$$\sin x - \sin y = 2 \cos \frac{x + y}{2} \sin \frac{x - y}{2}$$

$$\cos x + \cos y = 2 \cos \frac{x + y}{2} \cos \frac{x - y}{2}$$

$$\cos x - \cos y = -2 \sin \frac{x + y}{2} \sin \frac{x - y}{2}$$

****

$$\sin x \sin y = \frac{1}{2}[\cos(x - y) - \cos(x + y)]$$

$$\cos x \cos y = \frac{1}{2}[\cos(x - y) + \cos(x + y)]$$

$$\sin x \cos y = \frac{1}{2}[\sin(x - y) + \sin(x + y)]$$

Vrijednost funkcija za neke uglove:

α	$0°$	$\pi/6$ $30°$	$\pi/4$ $45°$	$\pi/3$ $60°$	$\pi/2$ $90°$	π $180°$	$3\pi/2$ $270°$	2π $360°$
$\sin\alpha$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0	-1	0
$\cos\alpha$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1	0	1
$\mathrm{tg}\,\alpha$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	nd	0	nd	0
$\mathrm{ctg}\,\alpha$	nd	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0	nd	0	nd

Veza između funkcija istog ugla:

	$\sin\alpha$	$\cos\alpha$	$\mathrm{tg}\,\alpha$	$\mathrm{ctg}\,\alpha$
$\sin\alpha$	$\sin\alpha$	$\pm\sqrt{1-\cos^2\alpha}$	$\pm\frac{\mathrm{tg}\,\alpha}{\sqrt{1+\mathrm{tg}^2\alpha}}$	$\pm\frac{1}{\sqrt{1+\mathrm{ctg}^2\alpha}}$
$\cos\alpha$	$\pm\sqrt{1-\sin^2\alpha}$	$\cos\alpha$	$\pm\frac{1}{\sqrt{1+\mathrm{tg}^2\alpha}}$	$\pm\frac{\mathrm{ctg}\,\alpha}{\sqrt{1+\mathrm{ctg}^2\alpha}}$
$\mathrm{tg}\,\alpha$	$\pm\frac{\sin\alpha}{\sqrt{1-\sin^2\alpha}}$	$\pm\frac{\sqrt{1-\cos^2\alpha}}{\cos\alpha}$	$\mathrm{tg}\,\alpha$	$\frac{1}{\mathrm{ctg}\,\alpha}$
$\mathrm{ctg}\,\alpha$	$\pm\frac{\sqrt{1-\sin^2\alpha}}{\sin\alpha}$	$\pm\frac{\cos\alpha}{\sqrt{1-\cos^2\alpha}}$	$\frac{1}{\mathrm{tg}\,\alpha}$	$\mathrm{ctg}\,\alpha$

Udaljenost tačaka $T_1(x_1, y_1)$ i $T_2(x_2, y_2)$:

$$d(T_1, T_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Polovište duži $\overline{T_1T_2}$:

$$P\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Vektori:

$$\overrightarrow{T_1T_2} = \vec{a} = (x_2 - x_1)\vec{i} + (y_2 - y_1)\vec{j} = a_1\vec{i} + a_2\vec{j}$$

Skalarni proizvod vektora:

$$\vec{a} \cdot \vec{b} = |\vec{a}| \cdot |\vec{b}| \cdot \cos\alpha \quad \vec{a} \cdot \vec{b} = a_1b_1 + a_2b_2$$

****

Jednačina pravca:

$$y - y_1 = k(x - x_1) \qquad k = \frac{y_2 - y_1}{x_2 - x_1}$$

Ugao φ između dvaju pravaca:

$$\operatorname{tg}\varphi = \frac{k_2 - k_1}{1 + k_1k_2}$$

Udaljenost tačke $T(x_1, y_1)$ od pravca $p: Ax + By + C = 0$:

$$d(T, p) = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$$

Krivulja drugoga reda	Jednačina	Tangenta u tački krivulje (x_1, y_1)
Kružnica središte $S(p,q)$	$(x-p)^2+(y-q)^2=r^2$	$(x_1-p)(x-p)+(y_1-q)(y-q)=r^2$
Elipsa fokusi $F_{1,2}(\pm e,0)$ $e^2=a^2-b^2$	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$	$\frac{x_1x}{a^2}+\frac{y_1y}{b^2}=1$
Hiperbola fokusi $F_{1,2}(\pm e,0)$ $e^2=a^2+b^2$ asimptote $y=\pm\frac{b}{a}x$	$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$	$\frac{x_1x}{a^2}-\frac{y_1y}{b^2}=1$
Parabola fokus $F\left(\frac{p}{2},0\right)$	$y^2=2px$	$y_1y=p(x+x_1)$

- Uvjet dodira pravca $y=kx+n$ i kružnice: $r^2\left(1+k^2\right)=(kp-q+n)^2$

****

Aritmetički niz:

$$a_n = a_1 + (n-1)\cdot d \qquad S_n = \frac{n}{2}(a_1 + a_n)$$

Geometrijski niz:

$$a_n = a_1 \cdot q^{n-1} \qquad S_n = a_1\frac{q^n - 1}{q - 1}$$

Geometrijski red:

$$S = \frac{a_1}{1-q};\ |q| < 1$$

****

Derivacija proizvoda:

$$(f \cdot g)' = f' \cdot g + f \cdot g'$$

Derivacija kvocijenta:

$$\left(\frac{f}{g}\right)' = \frac{f' \cdot g - f \cdot g'}{g^2}$$

Derivacija kompozicije:

$$(f \circ g)'(x) = f'(g(x)) \cdot g'(x)$$

Tangenta na graf funkcije f u $T(x_1, y_1)$:

$$y - y_1 = f'(x_1) \cdot (x - x_1)$$

Derivacije:

$$c' = 0, (x^n)' = n \cdot x^{n-1};\ n \neq 0$$

$$(\sin x)' = \cos x$$

$$(\cos x)' = -\sin x$$

$$(\operatorname{tg} x)' = \frac{1}{\cos^2 x}$$

Bilješke